Progress in Structural Engineering and Materials, Vol.2 No 4, 2000, pp. 502-507

Verification of Load Factors for Concrete Components Using Reliability and Optimisation Analysis – Background Documents for Implementing EUROCODES

Milan Holický[1] and Jana Marková[2]

[1]**Dr. Milan Holický**, Klokner Institute CTU, Šolínova 7, 166 08 Prague 6, Czech Republic, tel.: +420-2-2435 3842, fax: +420-2-2435 5232, E-mail: holicky@vc.cvut.cz.

[2] **Dr. Jana Marková**, Klokner Institute CTU, Šolínova 7, 166 08 Prague 6, Czech Republic, tel.: +420-2-24353501, E-mail: markova@klok.cvut.cz.

Acknowledgement: This study is a part of the research project CEZ: J04/98:210000029 "Risk Engineering and Reliability of Technical Systems", supported by funds of the Ministry of Education, Youth and Sports of Czech Republic.

Summary

The European standards for the design of construction works, the Structural Eurocodes, are presently at the stage of transformation from their ENV status into the operational documents EN. Available drafts and other working materials clearly indicate that the decision concerning alternative procedures and reliability elements will remain in the competency of national responsible authorities. One of the most important issues is the determination of the partial safety factors γ_G and γ_Q for permanent and variable loads. Simple examples of reinforced concrete components, considered in the present study, indicate that a structure designed using the recommended values for γ_G and γ_Q in the latest drafts of the proposed EN standards has an adequate reliability. Moreover, in some cases a foreseen reduction of γ_G and γ_Q may lead to unsatisfactory reliability. Thus it appears that a possible downward modification of recommended safety factors and other reliability elements should be supported by comprehensive studies and preferably limited to those cases, which are linked to national (regional) conditions, e.g. to climatic actions and their combinations. This recommendation is supported by probabilistic optimisation considering the total expected cost C_{tot} as an objective function and the partial factors γ_G and γ_Q of permanent and imposed loads as decisive parameters. The total cost C_{tot} further depends on the malfunction cost C_{f} to the marginal costs $C_{\text{m}G}$ and $C_{\text{m}Q}$. It appears, that the optimum values of the partial factors γ_G and γ_Q may be expected to be very close to the values recommended in the draft of EN 1990.

Keywords: reinforced concrete slab, column, characteristic load, partial safety factor, reliability index, probabilistic optimisation.

1. Introduction

Construction works in the whole Europe should soon be designed according to the harmonised procedures provided in the Eurocodes. In accordance with the aim of the European Committee for standardisation CEN/TC250, structural analyses and designs should differ only by numeric values of certain parameters as partial safety factors and characteristic values of climatic actions. However, the new harmonised standards attempt to eliminate even these differences, particularly in the regions located along the borders between Member States of CEN. Nevertheless it appears that the intended high degree of harmonisation decreases, and alternative procedures and different values of various reliability elements could be used, during the first introductory period of the new EN documents into the system of national regulations.

At present, transformation of the basic ENV documents for the design of construction works made of different materials is in its final stage. The Eurocodes will be soon implemented in the Member States of CEN as national standards together with appropriate National annexes. The national standards that are inconsistent with the new EN documents should be gradually modified or withdrawn. Obviously, this process may be complicated. Every Member State will have to make decision concerning the way of implementing the Eurocodes including the choice of the factors γ_G and γ_Q for the permanent and variable loads. The decision concerning the partial safety factors γ_G and γ_Q is one of the most important issues that should be specified in the National annexes.

The aim of this contribution, which is an extension of the previous study [1], is to analyse effects of the partial factors γ_G and γ_Q on reliability indices of basic reinforced concrete components – a slab and a column designed in accordance with the latest drafts of the prEN documents [2,3,4]. In the presented study one permanent and one variable action and the combination rule described in the latest draft of prEN 1990 [2] by the equation 6.10 is considered only. It follows, that the findings from this study should not be generalised for more complicated load cases and for other structural components made of different materials. It is expected and there is a need for further and more extensive studies to follow.

It should be mentioned that the Joint Nordic Group for Structural Matters published in 1999 an extensive report on reliability of various structural components [5] with a similar objective. The report presents an analysis of alternative procedures concerning the combination rules included in the prenormative document ENV 1991-1 (equation

9.10 and twin of equations 9.10 a and 9.10 b, now in the draft of prEN 1990 [2] equations 6.10, 6.10a and 6.10b) and verification of the partial safety factors for actions and materials. The obtained results indicate that structural components made of different materials and designed according to alternative procedures of the prenormative documents ENV have a significant variation in the reliability level. A possible verification of the partial factors γ_G and γ_Q for loads and the partial factors of material properties is indicated. A similar study on the probabilistic calibration of the partial safety factors was prepared in Finland [6]. Disturbing range in reliability of various structural components made of different materials indicate an urgent need for a comprehensive international analysis.

The submitted contribution attempts to extent the previous studies [5] and [6] by a brief reliability analysis of selected reinforced concrete components supplemented by optimisation analyses assuming the total cost as an objective function and considering the partial safety factors γ_G and γ_Q as decisive parameters.

2. Reinforced concrete slab

A simple supported reinforced concrete slab of 0,20 m depth and a span of 6 m is considered as a load-bearing component of a floor in the three different categories of loaded areas: residential, office and shopping area. The slab is made of concrete C 20/25 and reinforcing steel S 500. The design of the reinforced concrete slab follows the principles of the recent working drafts of the particular EN standards [2,3,4]. The design condition is considered by inequality

$$A_s f_{yk} / \gamma_s [d - 0,5\, A_s (f_{yk}/\gamma_s)/(f_{ck}/\gamma_c)] > (\gamma_G\, g_k + \gamma_Q\, q_k)\, L^2 / 8 \qquad (1)$$

where the common notation is used for the partial safety factors of materials properties

γ_s, γ_c and for the safety factors of permanent and variable actions γ_G and γ_Q. Additional notation is defined in Table 1. The reliability analysis is based on the following condition

$$K_R\, A_s f_y\, (d - 0{,}5\, A_s f_y / f_c) > K_E\, (\, g + q\,)\, L^2 / 8 \tag{2}$$

in which most of the basic variables are considered as random variables and the coefficients K_R and K_E describe model uncertainties for structural resistance R and action effects E.

3. Models of basic variables

Models of basic variables applied in the reliability analysis are listed in Table 1. Some of the models are assumed to be deterministic values, denoted “DET”, while the others are considered as random variables having the normal distribution “N”, the lognormal distribution “LN” or the gamma distribution “GAM”, as appropriate (in accordance to [7, 8 ,9]).

Table 1. Probabilistic models of basic variables.

Basic variable	Symbol	Name of basic variable	Distr. type	Unit	Char. value	Mean	Stand. dev.
Material properties	A_s	Reinforcement area	DET	m^2	nom	Nom	0
	f_c	Concrete strength	LN	MPa	20	30	5
	f_y	Yield strength	LN	MPa	435	560	30
Geometric data	L	Span of the slab	DET	m	6	6	0
	d	Effective height	N	m	0,17	0,17	0,012
Model [(1)] uncertainties	K_E	Uncertainty of load effect	N	-	1,0	1,0	0,10
	K_R	Uncertainty of resistance	N	-	1,0	1,0	0,07
Actions	g	Permanent load	N	MN/m^2	0,007	0,007	0,0007
	q	Imposed load	GAM	MN/m^2	See Table 2		

Note: [(1)] The model uncertainties cover effects of simplified models of action and resistance including inaccuracy in theoretical probabilistic models of basic variables.

Table 2 shows recommended characteristic values of the imposed load q_k [3], the partial safety factor γ_Q, required area of reinforcement A_s, mean μ_Q and standard deviation σ_Q of the model of imposed load and resulting reliability indices p_f and β for the three above-mentioned categories of loaded areas.

Table 2. Reliability indices (p_f, β) for 3 categories of loaded areas according to EN [3].

Loaded area	q_k [kN/m^2]	γ_Q	$A_s \times 10^4$ [m^2]	μ_Q [kN/m^2]	σ_Q [kN/m^2]	$p_f \times 10^5$	β
(A) Areas for residential activities	2,0	1,5	6,3	0,3	0,1	0,14	4,69
(B) Office areas	3,0	1,5	7,4	0,8	0,5 (1)	0,11	4,73
(D2) Shopping areas	5,0	1,5	9,6	1,0	0,9 (2)	0,22	4,59

Notes: (1) The standard deviation σ_Q corresponds to the loaded area of 24 m^2.

(2) The standard deviation σ_Q corresponds to the loaded area of 43 m^2.

4. Reinforced concrete column

A short reinforced concrete column, circular on plan, (made of the same concrete and reinforcement as the previous slab) having a cross-section with the diameter of 0,3 m and supporting a loaded office area of A = 35 m^2 (q_k = 3 kN/m^2) is considered as a second example. The column is loaded by permanent load having the characteristic value G_k = 0,560 MN and one imposed load Q_k = 0,105 MN. To simplify the following analysis, possible eccentricities are neglected. The design condition is given by inequality

$$A_s f_{yk}/\gamma_s + 0,8\,\pi\, d^2/4\ f_{ck}/\gamma_c > \gamma_G\, G_k + \gamma_Q\, Q_k \tag{3}$$

where again the common notation for the safety factors of material properties γ_s, γ_c and the safety factors for permanent and imposed loads γ_G and γ_Q are used. The required area of reinforcement is $A_s = 3,7 \times 10^{-4}$ m^2.

The reliability analysis is based on the following condition

$$K_R\,(A_s f_y + 0,8\,\pi\,d^2/4\;f_c) > K_E(\,G + Q\,). \tag{4}$$

Similarly as in the equation (2) K_R and K_E are the coefficients of model uncertainties of the structural resistance and the load effect. The models of basic variables used in the reliability analysis are the same as given in Table 1. The permanent load G is described by the normal distribution having the mean G_k and the coefficient of variation 0,1, the imposed load Q is described by the gamma distribution with the mean 0,028 MN and the standard deviation 0,0175 MN.

5. Results of reliability analysis

The probability of failure p_f and the reliability index β obtained from the reliability analysis of the reinforced concrete slab are listed in Table 2 and in Figures 1 and 2. The reliability index β for the slab of office areas for three different cases of the partial safety factors of imposed load γ_Q (from 1,3 to 1,7) is shown in Figure 1 as a function of the safety factor of permanent load γ_G.

The reliability index β for four values of the factor γ_G (from 1,0 to 1,35) as a function of the safety factor γ_Q is shown in Figure 2. It follows from Figure 2 that the reliability of the slab designed using the recommended partial factors $\gamma_{G,}$ γ_Q (1,35;1,5) of prEN 1990 [2] β = 4,73, and for assumed factors $\gamma_{G,}$ γ_Q (1,2;1,4) considered in some National application documents (see the point labelled ALT in Figure 2) to the ENV standards β = 4,18. The two figures clearly indicate, that the reliability index β is approximately linearly dependent on both safety factors γ_G and γ_Q.

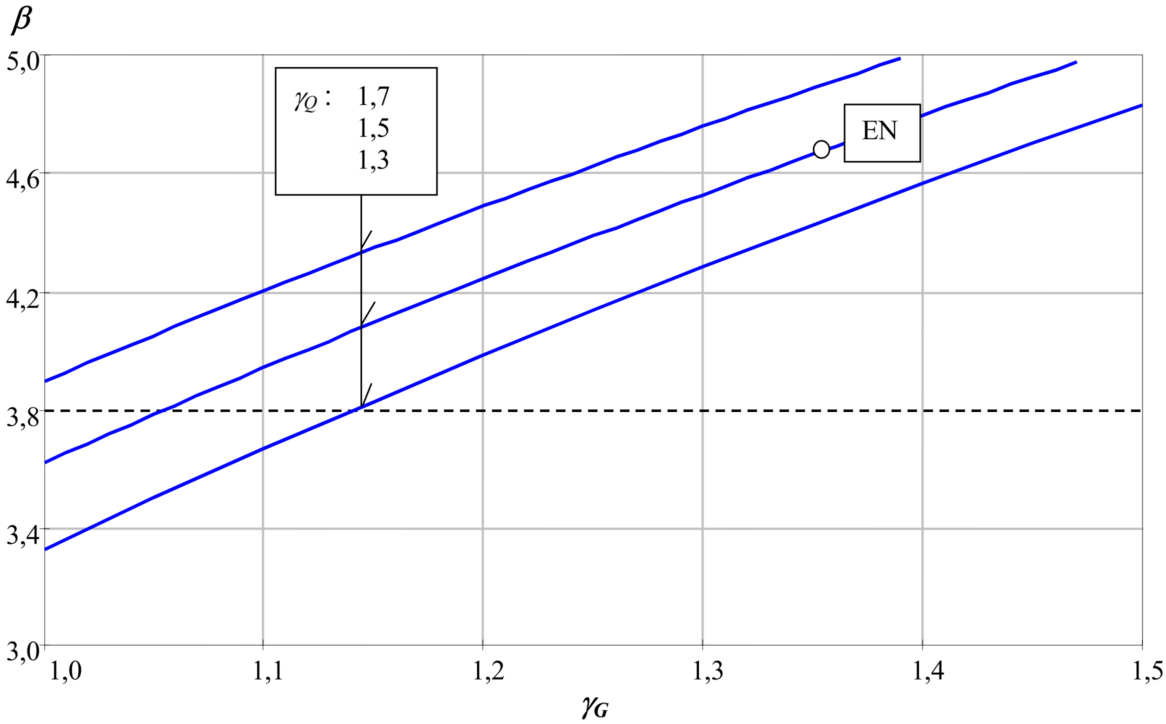

Figure 1: Reliability index β for the slab of office areas as a function of the safety factor of permanent load γ_G.

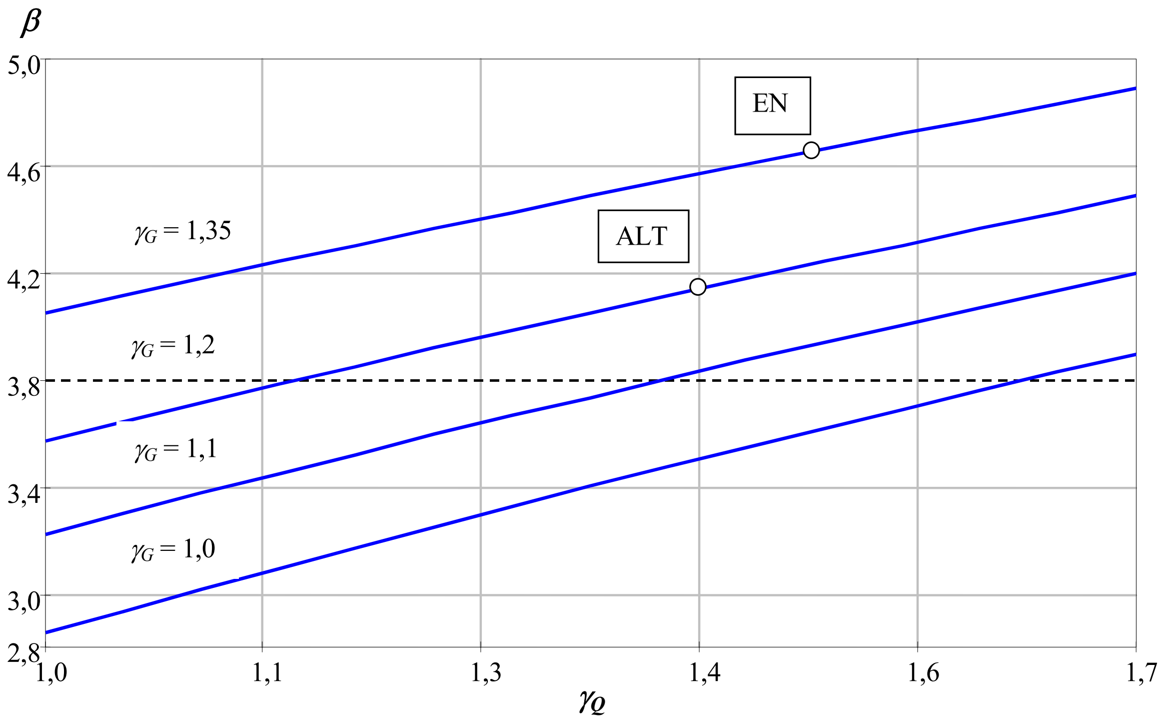

Figure 2: Reliability index β for the slab of office areas as a function of the safety factor of imposed load γ_Q.

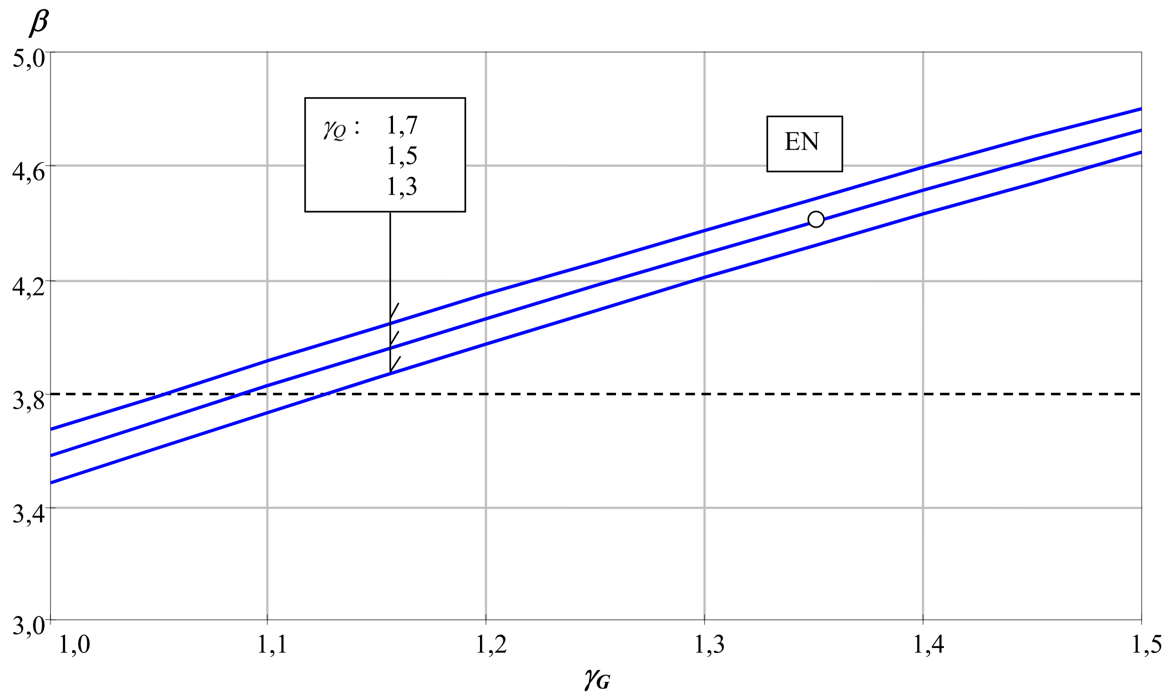

Figure 3: Reliability index β of the short column as a function of the safety factor of permanent load γ_G.

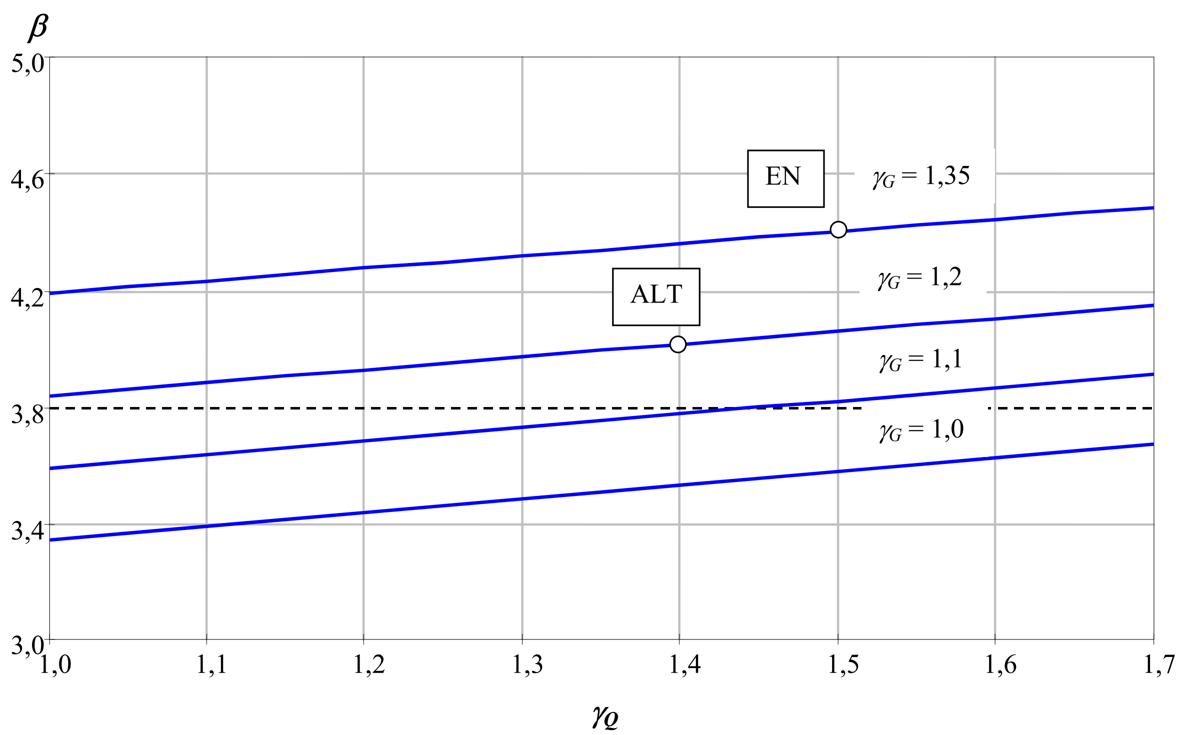

Figure 4: Reliability index β of the short column as a function of the safety factor of imposed load γ_Q.

Figures 3 and 4 show that similar result of the reliability analysis holds for the short column. The reliability index β of the column designed using the recommended safety factors γ_G = 1,35 and γ_Q = 1,5 is 4,41 (see the point labelled EN in Figure 4), considerably greater than the reliability index β = 4,03 of the column designed using the reduced partial factors γ_G = 1,2 and γ_Q = 1,4, (see the point labelled ALT in Figure 4). Moreover, the reliability index β = 4,03 seems to be very close to the recommended value 3,8. An adequate value of the reliability index may be found from the reliability optimisation analysis applied in the following paragraph.

6. Optimisation

The objective function is expressed as the total cost C_{tot} assumed as a sum of the initial cost C_0 of a component, of the marginal costs $\gamma_G\, C_{\text{m}G}$ and $\gamma_Q\, C_{\text{m}Q}$, where γ_G and γ_Q denotes the partial safety factors of the self-weight and the imposed load, $C_{\text{m}G}$ and $C_{\text{m}Q}$ being costs per increment of γ_G and γ_Q, and an expected cost of internal failure of a component (loss of its capacity) given as $p_{\text{f}}(\gamma_G, \gamma_Q)C_{\text{f}}$, where $p_{\text{f}}(\gamma_G, \gamma_Q)$ is a probability of a component failure corresponding to condition (2) and C_{f} is the malfunction cost given the failure occurs. The expected total cost C_{tot} is thus

$$C_{\text{tot}} = C_0 + \gamma_G\, C_{\text{m}G} + \gamma_Q\, C_{\text{m}Q} + p_{\text{f}}(\gamma_G, \gamma_Q)\, C_{\text{f}} \tag{5}$$

It is assumed that all costs C_0, $C_{\text{m}G}$, $C_{\text{m}Q}$ and the failure cost C_{f} are independent on the partial safety factors γ_G, γ_Q. Necessary conditions for the minimum total cost may be then expressed as

$$C_{\text{m}G} / C_{\text{f}} = -\frac{\partial\, p_{\text{f}}(\gamma_G;\gamma_Q)}{\partial \gamma_G}, \quad C_{\text{m}Q} / C_{\text{f}} = -\frac{\partial\, p_{\text{f}}(\gamma_G;\gamma_Q)}{\partial \gamma_Q} \tag{6}$$

The partial derivatives (6) may be used to find precise values of the optimum decisive parameters γ_G, γ_Q. However, for the first assessment it is sufficient to consider the partial (comparative) cost $\gamma_Q + p_f(\gamma_G;\ \gamma_Q)\ C_f /\ C_{mQ}$ that is shown in Figure 5 for γ_G = 1,35 and selected values C_f / C_{mQ}. These results have been obtained using the software programme COMREL (1997).

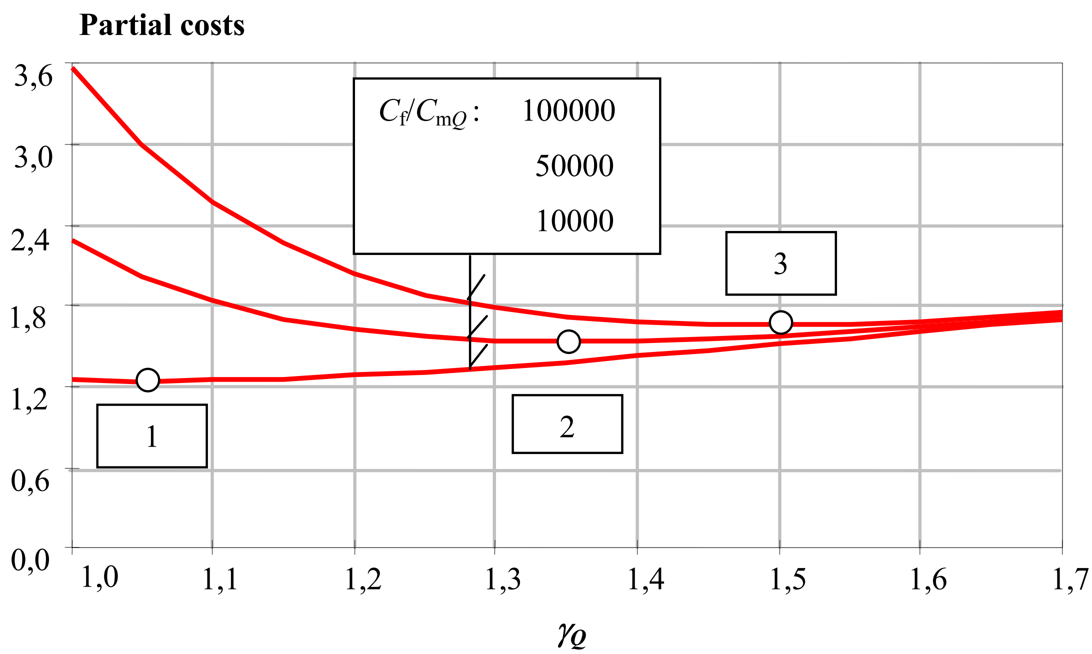

Figure 5: The total cost C_{tot} as a function of the safety factor γ_Q for γ_G = 1,35 and the selected ratios C_f/C_{mQ} (the optimum factors γ_Q correspond to the points 1,2 and 3).

It follows from Figure 5 that for C_f/C_{mQ} = 10000, which seems to be a rather low value, the optimum partial safety factor γ_Q is γ_Q = 1,05, for the ratio C_f/C_{mQ} = 50000 the safety factor is γ_Q = 1,35 and for C_f/C_{mQ} = 100000 the optimum factor γ_Q = 1,5.

Similarly the partial cost $\gamma_G + p_f(\gamma_G;\ \gamma_Q)\ C_f /\ C_{mG}$ for γ_Q = 1,50 and the selected values of C_f / C_{mG} are shown in Figure 6.

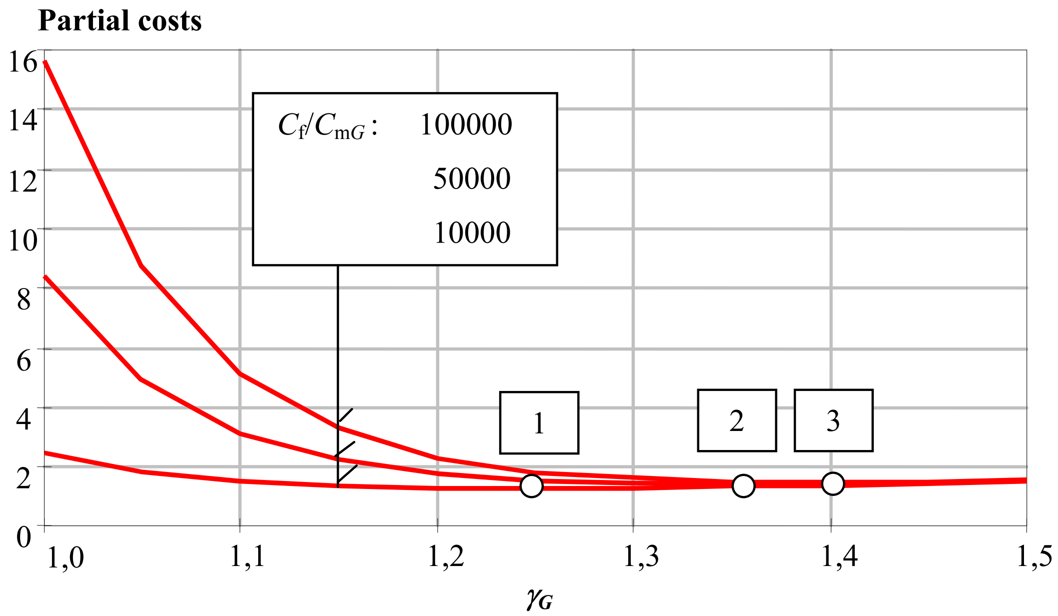

Figure 6: The total cost C_{tot} as a function of the safety factor γ_G for $\gamma_Q = 1{,}50$ and the selected ratios C_f/C_{mG} (the optimum factors γ_G correspond to the points 1,2 and 3).

It follows from Figure 6 that for the ratio $C_f/C_{mG} = 10000$, which seems to be a rather low ratio, the optimum value of partial safety factor γ_G is $\gamma_G = 1{,}25$, for the ratio $C_f/C_{mG} = 50000$ the safety factor is $\gamma_G = 1{,}35$ and for the ratio $C_f/C_{mQ} = 100000$ the optimum factor $\gamma_G = 1{,}4$.

It should be however noted that the optimisation model used in this study is a common simplification of actual conditions and should be considered as a first insight into economic consequences of possible changes of the partial safety factors γ_G and γ_G. Moreover, the indicated cost data have no rational background and should be considered as a result of subjective judgement only. Research is required so as a more detailed analysis taking into account more realistic optimisation model and cost data may be made.

7. Concluding remarks

Although an excellent initiative to develop a unified set of technical documents within European countries for the design of various construction works made of different materials seems to be an extremely complicated and difficult task. The expectancy of a high degree of harmonisation, has been gradually decreased and alternative design procedures and reliability elements seem to be now allowed in individual countries. This is also due to the fact that safety remains a National responsibility. Moreover several technical questions are not yet solved and will certainly become subjects of further discussions during the co-existence of the newly developing Eurocodes and the national standards. Further harmonisation based on national and international studies is therefore expected within the medium term.

This contribution indicates that modification (reduction) of the partial safety factors γ_G and γ_Q recommended in EN should be thoroughly examined before their introduction into National annexes of the EN standards. It is shown that the reliability of reinforced concrete components may be significantly decreased when using reduced partial factors γ_G and γ_Q. The partial safety factors γ_G = 1,35 and γ_Q = 1,5 recommended in the Eurocodes seem to represent a reasonable optimum for the wide range of expected malfunction costs. Thus, unless comprehensive and conclusive studies are available, recommendations provided in prEN 1990 should be accepted.

Further reliability analysis and probabilistic optimisations of various structural components made of different materials and exposed to various load effects or their combinations are needed. An international co-operation in this area will be extremely useful. The aim of these studies will be to specify the optimal national and international recommendations for alternative design procedures, categorisation of structures and safety elements (e.g. the partial safety factors for load effects, for material properties, target values of the reliability indices). It is also expected that results from studies will subsequently make it possible to reach the desired increased degree of harmonisation of the European standards for construction works.

References:

1. Holický M & Marková J. Reliability analysis of a continuous reinforced concrete slab designed according to Eurocodes. In: Proceedings of the International Conference Concrete and Concrete Structures. Košice. SK. 1998. 188-193.
2. Draft prEN 1990 Basis of design. CEN/TC 250 working material. 2000.
3. Draft prEN 1991-1-1 Actions on structures. Densities, self-weight, and imposed loads on buildings. CEN/TC 250/SC1 working material. 2000.
4. Draft prEN 1992-1 Design of concrete structures. General rules and rules for buildings. CEN/TC 250/SC2 working material. 2000.
5. SAKO; Joint Committee of NKB and INSTA-B. Basis of Design of Structures. Proposal for Modification of Partial Safety Factors in Eurocodes. 1999.
6. Ministry of the Environment, Housing and Building Department, Finland. Probabilistic Calibration of Partial Safety Factors (Eurocode and Finish proposal). 2000.
7. CIB Report: Action on Structures, Self-weight Loads. Report 115, 1989.
8. CIB Report: Action on Structures, Live Loads in Buildings. Report 116, 1989.
9. JCSS - Joint Committee on Structural Safety: Probabilistic Model Code, working materials, 1999 and 2000.